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P 10.5-27 Determine the steady state voltage, \( v(t) \), for each of these circuits:

\begin{align*}
\text{(a)} \quad & 80 \Omega + 20 \Omega \\
& 40 \Omega + 100 \Omega \\
& + v(t) - \\
& 24 \text{ V} \\
\end{align*}

\begin{align*}
\text{(b)} \quad & 3 \text{ H} + 5 \text{ mF} \\
& 20 \Omega + 25 \Omega \\
& + v(t) - \\
& 24 \cos (20t + 45^\circ) \text{ V} \\
& + 4 \text{ H} + 4 \text{ mF} \\
& 2 \text{ mF} + 20 \Omega + 15 \Omega \\
\end{align*}

**Solution:**
(a) Using voltage division twice

\[
v(t) = \frac{40}{40+80} \times 24 - \frac{100}{20+100} \times 24 = -12 \text{ V}
\]
(b) Represent the circuit in the frequency domain using phasors and impedances.

\[
\begin{align*}
\mathbf{Z}_1 &= 20 \, \Omega \\
\mathbf{Z}_2 &= j(20)4 + \left( \frac{1}{j(20)(0.002)} \right) 20 = 12.2 + j70.2 = 71.30 \angle 80.2^\circ \, \Omega \\
\mathbf{Z}_3 &= j(20)3 + \frac{1}{j(20)(0.005)} + 25 = 25 + j50 = 55.90 \angle 63.4^\circ \, \Omega \\
\mathbf{Z}_4 &= \frac{1}{j(20)(0.004)} + 15 = 15 - j12.5 = 19.53 \angle -39.8^\circ \, \Omega
\end{align*}
\]

Using voltage division twice

\[
\mathbf{V} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \times 24 \angle 45^\circ - \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} \times 24 \angle 45^\circ = 24.8 \angle 80^\circ \, V
\]

so

\[
\mathbf{v}(t) = 24.8 \cos (20t + 80^\circ) \, V
\]
P 10.6-5  A commercial airliner has sensing devices to indicate to the cockpit crew that each door and baggage hatch is closed. A device called a search coil magnetometer, also known as a proximity sensor, provides a signal indicative of the proximity of metal or other conducting material to an inductive sense coil. The inductance of the sense coil changes as the metal gets closer to the sense coils. The sense coil inductance is compared to a reference coil inductance with a circuit called a balanced inductance bridge (see Figure P 10.6-5). In the inductance bridge, a signal indicative of proximity is observed between terminals a and b by subtracting the voltage at b, $v_b$, from the voltage at a, $v_a$ (Lenz, 1990).

The bridge circuit is excited by a sinusoidal voltage source $v_s = \sin (800 \pi t)$ V. The two resistors, $R = 100 \ \Omega$, are of equal resistance. When the door is open (no metal is present), the sense coil inductance, $L_S$, is equal to the reference coil inductance, $L_R = 40$ mH. In this case, what is the magnitude of the signal $V_a - V_b$?

When the airliner door is completely closed, $L_S = 60$ mH. With the door closed, what is the phasor representation of the signal $V_a - V_b$?

Solution:

$$v_s = \sin (2\pi \cdot 400t) \ V$$

$$R = 100 \ \Omega$$

$$L_R = 40 \text{ mH}$$

$$L_S = \begin{cases} 
 40 \text{ mH} & \text{door opened} \\
 60 \text{ mH} & \text{door closed}
\end{cases}$$
With the door open  \[|V_A - V_B| = 0\] since the bridge circuit is balanced.

With the door closed  \[Z_{ts} = j(800 \pi)(0.04) = j100.5 \ \Omega\] and \[Z_{ts} = j(800 \pi)(0.06) = j150.8 \ \Omega\]

The node equations are:

KCL at node B:\[\frac{V_B - V_c}{R} + \frac{V_B}{Z_{ts}} = 0 \Rightarrow V_B = \frac{j100.5}{j100.5 + 100} V_c\]

KCL at node A:\[\frac{V_A - V_c}{R} + \frac{V_A}{Z_{ts}} = 0\]

Since \[V_c = |V_s| = 1 \ V\] \[V_B = 0.709 \angle 44.86^\circ \ V\] and \[V_A = 0.833 \angle 33.55^\circ \ V\]

Therefore

\[V_A - V_B = 0.833 \angle 33.55^\circ - 0.709 \angle 44.86^\circ = (0.694 + j.460) - (0.503 + j0.500) = 0.191 - j0.040 \]
\[= 0.195 \angle -11.83^\circ \ V\]
P 10.6-12 Determine the node voltage at nodes a and b in each of these circuits:

![Circuit Diagram](image)

**Solution**

(a)

The node equations are

\[ \frac{24 - v_a}{40} = \frac{v_a - v_b}{20} + \frac{v_a}{15} \]
\[ \frac{24 - v_b}{25} + \frac{v_a - v_b}{20} = \frac{v_b}{50} \]

or

\[
\begin{bmatrix}
\frac{1}{40} & \frac{1}{20} & \frac{1}{15} & -\frac{1}{20} \\
-\frac{1}{20} & \frac{1}{25} & \frac{1}{20} & \frac{1}{50}
\end{bmatrix}
\begin{bmatrix}
v_a \\
v_b
\end{bmatrix}
= \begin{bmatrix}
\frac{24}{40} \\
\frac{24}{25}
\end{bmatrix}
\]

Solving using MATLAB gives \( v_a = 8.713 \text{ V} \) and \( v_b = 12.69 \text{ V} \)
(b) Use phasors and impedances to represent the circuit in the frequency domain as

\[ \begin{align*}
Z_1 &= 25 + j(20)4 = 25 + j80 = 83.82 \angle 72.7^\circ \, \Omega \\
Z_2 &= \left( 40 \left[ \frac{1}{j(20)(0.004)} \right] \right) + j(20)5 = 3.56 + j88.6 = 88.68 \angle 87.7^\circ \, \Omega \\
Z_3 &= 20 \, \Omega \\
Z_4 &= 15 + j(20)2 = 15 + j40 = 42.72 \angle 69.4^\circ \\
Z_5 &= j(20)3 + \frac{1}{j(20)(0.005)} = j50 = 50 \angle 90^\circ \, \Omega
\end{align*} \]

where

\[ \begin{align*}
\frac{24 \angle 45^\circ - V_a}{Z_2} &= \frac{V_a + V_a - V_b}{Z_4 + Z_5} \\
\frac{24 \angle 45^\circ - V_b}{Z_1} + \frac{V_a - V_b}{Z_3} &= \frac{V_b}{Z_4}
\end{align*} \]

The node equations are

\[ \begin{bmatrix}
\frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4} \\
\frac{-1}{Z_3} \\
\frac{1}{Z_1} + \frac{1}{Z_5} + \frac{1}{Z_3}
\end{bmatrix} \begin{bmatrix}
V_a \\
V_b
\end{bmatrix} = \begin{bmatrix}
\frac{24 \angle 45^\circ}{Z_2} \\
\frac{24 \angle 45^\circ}{Z_1}
\end{bmatrix} \]

Solving using MATLAB gives

\[ \begin{align*}
V_a &= 7.89 \angle 44.0^\circ \\
V_b &= 8.45 \angle 45.1^\circ
\end{align*} \]

So

\[ v_a(t) = 7.89 \cos \left( 20t + 44^\circ \right) \, V \]

\[ v_b(t) = 8.45 \cos \left( 20t + 45.1^\circ \right) \, V \]
P 10.6-14 Determine the voltage \( v_o(t) \) when \( v_s(t) = 25 \cos (100t - 15^\circ) \) V.

\[ \text{Solution:} \]

Represent the circuit in the frequency domain using impedances and phasors.

\[
\begin{align*}
25 \angle -15^\circ & \quad \text{V} \\
\frac{I}{5 \, \Omega} & \quad j500 \, \Omega \\
\frac{9 \, I}{10 \, \Omega} & \quad 40 \, \Omega
\end{align*}
\]

The mesh currents are \( I \) and \( 10I \). Apply KVL to the supermesh corresponding to the dependant current source to get

\[
(j500)I + (-j5)(10I) + 40(10I) - 25 \angle -15^\circ = 0
\]

So

\[
I = \frac{25 \angle -15^\circ}{400 + j450} = 0.04152 \angle -63.37^\circ \, \text{A}
\]

The output voltage is

\[
V = 40(10I) = 16.61 \angle -63.37^\circ \, \text{V}
\]

So

\[
v(t) = 16.61 \cos(100t - 63.37^\circ) \, \text{V}
\]
The circuit shown in Figure P 10.6-16 has two inputs:

\[ v_1(t) = 50 \cos (20t - 75^\circ) \text{ V} \]

\[ v_2(t) = 35 \cos (20t + 110^\circ) \text{ V} \]

When the circuit is at steady state, the node voltage is

\[ v(t) = 21.25 \cos (20t - 168.8^\circ) \text{ V} \]

Determine the values of \( R \) and \( L \).

**Solution:**

Represent the circuit in the frequency domain using phasors and impedances. Apply KCL at the top node of \( R \) and \( L \) to get

\[
\frac{(50 \angle -75^\circ) - V}{j40} + \frac{35 \angle 100^\circ - V}{40} = \frac{V}{R \angle j\omega L}
\]

\[
\Rightarrow \frac{50 \angle -75^\circ}{40 \angle 90^\circ} + \frac{35 \angle 110^\circ}{40} = \left( \frac{1}{j40} + \frac{1}{40} + \frac{1}{R} - j \frac{1}{20L} \right) V
\]

Using the given equation for \( v(t) \) we get

\[
21.25 \angle -168.8^\circ = V = \frac{1.587 \angle 161.7^\circ}{0.025(1 - j) + \frac{1}{R} - j \frac{1}{20L}}
\]

Then

\[
\frac{1}{R} - j \frac{1}{20L} = \frac{1.587 \angle 161.7^\circ}{21.25 \angle -168.8^\circ} - 0.025(1 - j) = 0.04 - j0.01176
\]

Finally,

\[
R = \frac{1}{0.04} = 25 \text{ \Omega} \quad \text{and} \quad L = \frac{1}{20(0.01176)} = 4.25 \text{ H}
\]
P 10.6-19 Determine the steady state voltage $v_o(t)$:

![Circuit Diagram]

**Solution:**
Represent the circuit in the frequency domain using phasors and impedances.

\[
\begin{align*}
\frac{20\angle 0^\circ - V}{j40} &= \frac{V}{25} + \frac{V - 5V}{-j20} \\
\frac{5V - V_o}{10} &= \frac{V_o}{-j10}
\end{align*}
\]

The node equations are

\[
\begin{bmatrix}
\frac{1}{25} - j \frac{1}{5} - j \frac{1}{40} & 0 \\
-\frac{1}{2} & \frac{1}{10} + j \frac{1}{10}
\end{bmatrix}
\begin{bmatrix}
V \\
V_o
\end{bmatrix}
= 
\begin{bmatrix}
-j0.5 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.04 - j0.225 & 0 \\
-0.50 & 0.10 + j0.10
\end{bmatrix}
\begin{bmatrix}
V \\
V_o
\end{bmatrix}
= 
\begin{bmatrix}
-j0.5 \\
0
\end{bmatrix}
\]

Solving gives $V = 2.188 \angle -10.1^\circ$ V and $V_o = 7.736 \angle -55.1^\circ$ V

So $v_o(t) = 7.736\cos(5t - 55.1^\circ)$ V
P10.7-2 Determine the Thevenin equivalent of this circuit when $v_s(t) = 5 \cos (4000t-30^\circ)$ V.

Solution:

Find $V_{oc}$:

$$V_{oc} = (5 \angle -30^\circ) \left(\frac{80 + j80}{80 + j80 - j20}\right)$$

$$= (5 \angle -30^\circ) \left(\frac{80 \sqrt{2} \angle -45^\circ}{100 \angle 36.9^\circ}\right)$$

$$= 4 \sqrt{2} \angle -21.9^\circ \text{ V}$$

Find $Z_t$:

$$Z_t = \frac{(-j20)(80 + j80)}{-j20 + 80 + j80} = 23 \angle -81.9^\circ \Omega$$

The Thevenin equivalent is
**P10.7-4** Determine the Thevenin equivalent of this circuit when \( v_s(t) = 10 \cos (10,000t + 53.1^\circ) \) V.

![Circuit Diagram]

**Solution:**

First, determine \( V_{oc} \):

The node equation is:

\[
\frac{V_{oc}}{-j4} + \frac{V_{oc}-(6+j8)}{j2} = \frac{3}{2} \left( \frac{V_{oc}-(6+j8)}{j2} \right) = 0
\]

\( V_{oc} = 3 + j4 = 5 \angle 53.1^\circ \) V

Next, determine \( I_{sc} \):

The node equation is:

\[
\frac{V}{2} + \frac{V}{-j4} + \frac{V-(6+j8)}{j2} = \frac{3}{2} \left[ \frac{V-(6+j8)}{j2} \right] = 0
\]

\( V = \frac{3 + j4}{1 - j} \) then \( I_{sc} = \frac{V}{2} = \frac{3 + j4}{2 - j2} \)

The Thevenin impedance is \( Z_t = \frac{V_{oc}}{I_{sc}} = 3 + j4 \left( \frac{2-j2}{3+j4} \right) = 2 - j2 \) Ω

The Thevenin equivalent is
Consider the circuit of Figure P 10.7-6, where we wish to determine the current $I$. Use a series of source transformations to reduce the part of the circuit connected to the 2-Ω resistor to a Norton equivalent circuit, and then find the current in the 2-Ω resistor by current division.

**Solution:**

$$Z_1 = \frac{(-j3)(4)}{-j3+4} = 2.4 \angle -53.1^\circ \, \Omega$$

$$= 1.44 - j1.92 \, \Omega$$

$$Z_2 = Z_1 + j4$$

$$= 1.44 + j2.08$$

$$= 2.53 \angle 55.3^\circ \, \Omega$$

$$Z_3 = 3.51 \angle -37.9^\circ \, \Omega$$

$$= 2.77 - j2.16 \, \Omega$$

$$I = (2.85 \angle -78.4^\circ) \left( \frac{3.51 \angle -37.9^\circ}{2.77 - j2.16 + 2} \right) = (2.85 \angle -78.4^\circ) \left( \frac{3.51 \angle -37.9^\circ}{5.24 \angle -24.4^\circ} \right) = 1.9 \angle -92^\circ \, \text{A}$$
P10.7-8 Determine the Thevenin equivalent of the circuit in (a):

\[ V_1 = \frac{j10}{8+j10} \cdot 5e^{-j90} = 3.9e^{-j51} \]
\[ V_2 = \frac{j20}{j20 - j2.4} \cdot 5e^{-j90} = 5.68e^{-j90} \]
\[ V_t = V_1 - V_2 = 3.9e^{-j51} - 5.68e^{-j90} = 3.58e^{j47} \]

\[ Z_t = \frac{8(j10) + \frac{-j2.4(j20)}{-j2.4 + j20}}{8 + j10} = 4.9 + j1.2 \]
P 10.8-5  The input to the circuit shown in Figure P 10.8-5 is the current source current
\( i_s(t) = 36 \cos (25t) + 48 \cos (50t + 45^\circ) \) mA

Determine the steady-state current, \( i(t) \).

Solution:
Use superposition in the time domain. Let
\[
\left\{ \begin{array}{c}
i_{s1}(t) = 36 \cos (25t) \text{ mA} \\
i_{s2}(t) = 48 \cos (50t + 45^\circ) \text{ mA}
\end{array} \right.
\]

We will find the response to each of these inputs separately. Let \( i_i(t) \) denote the response to \( i_{si}(t) \) for \( i = 1,2 \). The sum of the two responses will be \( i(t) \), i.e.
\[
i(t) = i_1(t) + i_2(t)
\]

Represent the circuit in the frequency domain as

Use KVL to get
\[
V_i = Z_1 I_i - 4I_i
\]

Apply KCL to the supernode corresponding to the dependent voltage source.
\[
I_{si} = I_i + \frac{V_i}{Z_2} = \frac{Z_1 + Z_2 - 4}{Z_2} I_i
\]

Or,
\[
I_i = \frac{Z_2 I_{si}}{Z_1 + Z_2 - 4}
\]
Consider the case \( i = 1 : i_{s1}(t) = 26\cos(25t) \) mA.

Here \( \omega = 25 \text{ rad/s} \) and

\[
I_s = 36\angle 0^\circ \text{ mA}
\]

\[
Z_1 = 20 + \frac{1}{j(25)(0.002)} = 20 - j20 \ \Omega
\]

\[
Z_2 = j50 + \left( 15 + \frac{1}{j(25)(0.004)} \right) = 43.3\angle 83.9^\circ \ \Omega
\]

and

\[
I_1 = 50.4\angle 35.7^\circ \text{ mA}
\]

so

\[
i(t) = 50.4\cos(25t + 35.7^\circ) \text{ mA}
\]

Next consider \( i = 2 : i_{s2} = 48\cos(50t + 45^\circ) \) mA.

Here \( \omega = 50 \text{ rad/s} \) and

\[
I_{s2} = 48\angle 45^\circ \text{ mA}
\]

\[
Z_1 = 20 + \frac{1}{j(50)(0.002)} = 20 - j10 \ \Omega
\]

\[
Z_2 = j100 + \left( 15 + \frac{1}{j(50)(0.004)} \right) = 95.5\angle 89.1^\circ \ \Omega
\]

(Notice that \( Z_1 \) and \( Z_2 \) change when \( \omega \) changes.)

\[
I_2 = 52.5\angle 55.7^\circ \text{ mA}
\]

so

\[
i_2(t) = 52.5\cos(50t + 55.7^\circ) \text{ mA}
\]

Finally, using superposition in the time domain gives

\[
i(t) = 50.4\cos(25t + 35.7^\circ) + 52.5\cos(50t + 55.7^\circ) \text{ mA}
\]
**P10.8-9** Determine the current $i(t)$ for this circuit when $v_1(t)=10\cos(10t)\text{ V}$

![Circuit Diagram]

**Solution:**

Use superposition. First, find the response to the voltage source acting alone:

$$Z_{eq} = \frac{-j10 \cdot 10}{10 - j10} = 5(1 - j)\ \Omega$$

Replacing the parallel elements by the equivalent impedance. The write a mesh equation:

$$-10 + 5I_1 + j15I_1 + 5(1 - j)I_1 = 0 \Rightarrow I_1 = \frac{10}{10 + j10} = 0.707 \angle -45^\circ \text{ A}$$

Therefore:

$$i_1(t) = 0.707 \cos(10t - 45^\circ) \text{ A}$$

Next, find the response to the dc current source acting alone:

Current division: $I_2 = \frac{-10}{15} \times 3 = -2 \text{ A}$

Using superposition:

$$i(t) = 0.707 \cos(10t - 45^\circ) - 2 \text{ A}$$